Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

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Mid Term Exam#2, Spring 2020

Name and SR.No:

Instructions:

- This is a take home exam. You are allowed to refer to anything, including online materials; do cite them. However, there is absolutely no collaboration with any one, be it a human being or an AI bot.
- This exam is assigned on Saturday, May 9th, 2020 at 6 pm. You must turn in your scanned or latexed solutions by May 14th, 2020, 6 pm in one pdf file with all the necessary detailed calculations and software code in an Appendix for credit.
- There are three main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and justification for partial credit.
- Make any reasonable assumptions if really required.
- Do not panic, do not cheat.
- Good luck!

Question No.	Points scored
1	
2	
3	
Total points	

PROBLEM 1: Let us try to construct a different scaling function and study the properties. Consider

$$\theta(t) = \begin{cases} 0 & t \leq 0, \\ 1 & t \geq 1. \end{cases}$$

In the interval $0 \leq t \leq 1$, let $\theta(t)$ be such that $\theta(t) + \theta(1-t) = 1$. Let us choose one such $\theta(t)$ as

$$\theta(t) = \begin{cases} 0 & t \leq 0, \\ at^p + bt^{p+1} & 0 \leq t \leq 1, \\ 1 & t \geq 1. \end{cases}$$

where a and b are integers, and $p > 1$ is the smallest positive integer satisfying $\theta(t) + \theta(1-t) = 1$.

(1) What choices of a, b, p satisfy $\theta(t)$?
(2) From first principles, for the choice of your solution from earlier part, examine if the scaling function whose frequency domain response is defined by

$$\phi(\omega) = \begin{cases} \sqrt{\theta\left(-b + \frac{a\omega}{2\pi}\right)} & \omega \leq 0, \\ \sqrt{\theta\left(-b - \frac{a\omega}{2\pi}\right)} & \omega \geq 0, \end{cases}$$

satisfies multiresolution analysis.
(3) Sketch $\phi(t)$ and $\phi(\omega)$.
(4) Using a computer code, numerically evaluate the time-frequency uncertainty relationship for this scaling function. How does this compare to a Haar scaling function in the zeroth scale?

(45 pts.)

PROBLEM 2: Consider all points lying uniformly distributed within a quadrilateral whose corners are given by $(1,0)$, $(1,2)$, $(2,1)$ and $(3,4)$.

(1) Obtain the Karhunen Loeve representation of the points analytically.
(2) Verify your theoretical results through an algorithm by writing a software code to generate the points randomly satisfying the 2D distribution. Plot the principal directions.
(3) If you were to retain the dominant direction, how much of energy is lost during dimensionality reduction? Compute your results analytically and verify it experimentally.

(35 pts.)

PROBLEM 3: While trying to isolate and null out a spike, three course students adopted the following approaches. One student used wavelets to decompose the signal using an appropriate Haar basis to the necessary scale required and then claimed to null the spike out. The second student applied an overlapping sliding window over the signal in time, going sequentially from the start of the signal, tried to analyze any sharp changes in the signal, applied a threshold and claimed to null the spike. The third student did essentially what the second student did but applied a non-overlapping window and claimed to null the spike out. Which approach do you prefer, reason why? Examine the pros and cons of the various proposals and comment on the proposed solutions in terms of the *efficacy* of the signal processing techniques and the complexity of the algorithms.

(20 pts.)