Indian Institute of Science

Mathematical Methods and Techniques in Signal Processing

Instructor: Shayan Srinivasa Garani
Home Work #4, Spring 2020

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, d = # days late

Assigned date: Apr. 16th 2020 **Due date:** Apr. 30th 2020

PROBLEM 1:

Consider a 1D Haar wavelet filter bank with continuous time filters and dyadic representation over J stages i.e., you will have $J + 1$ parallel branches in the filter bank. Starting with a Haar mother filter over a duration of 1 time unit, obtain the time-frequency uncertainty relation at each stage of the filterbank as a function of increasing time resolution. You need to arrive at the general formula as a function of the scale. Derive and sketch the overall frequency domain responses of the analysis and the synthesis filters in each branch of the filter bank. (20 pts.)

PROBLEM 2:

Consider all 2D points lying uniformly distributed inside an ellipse centered at the origin, with major and minor axes lengths $2a$ and $2b$ respectively. We would like to collapse the 2D points to 1D using dimensionality reduction. How would you do this? Show all your analytical steps carefully. Validate your results through a software code, implementing the relevant equations. Verify your results by simulating the scenario over large number of finite data points and appropriate transformations. (20 pts.)

PROBLEM 3: This question has several sub-parts.

(1) All square integrable functions are absolutely integrable. True or false? Justify.
(2) The signal $s(t) = t \sin(\frac{1}{t})$ has a Fourier series over the cyclically repeated interval $-\frac{1}{\pi} < t < \frac{1}{\pi}$. True or false? Justify. What about the signal $s(t) = \tan(t)$ over the real line? (10 pts.)