Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

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Mid Term Exam#1, Spring 2020

Name and SR.No:

Instructions:

- You are allowed my lecture handouts and your class notes and nothing else.
- The time duration is 3 hrs.
- There are four main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and justification for partial credit.
- Make any reasonable assumptions if really required.
- Do not panic, do not cheat.
- Good luck!

Question No.	Points scored
1	
2	
3	
4	
Total points	

PROBLEM 1: You need to write short answers for the following.

(1) Discrete time sequences can have aperiodic spectrum. True or false. Justify. (5 pts.)

(2) Let $\mathbf{X} = [X[0], X[1], ..., X[N-1]]^T$ be the N point DFT of $\mathbf{x} = [x[0], x[1], ..., x[N-1]]^T$. Explicitly show the structure of $\mathbf{T}$, where $\mathbf{X} = \mathbf{T}\mathbf{x}$. (5 pts.)

(3) How many modes does $s(t) = \tan(\omega t)$ have? (5 pts.)

(4) You observe N linear and time-invariant (LTI) systems that have identical magnitude responses. However, exactly one of them is minimum phase. You are not told anything about the parameters of the model, except allowed to drive input signals through the system and measure the outputs on a scope. As a DSP engineer, can you decipher which of the N systems is minimum phase by devising a test? Justify your answer carefully. (5 pts.)

PROBLEM 2: This problem has 3 parts.

(1) For any two vectors $\bar{x}$ and $\bar{y}$ in a vector space, show that $\langle \bar{x}, \bar{y} \rangle = \frac{1}{4}(||\bar{x}+\bar{y}||^2 - ||\bar{x}-\bar{y}||^2)$. (5 pts.)

(2) Let $X = L_2[-\pi, \pi]$. Let $S_1 = \text{span}(1, \cos(t), \cos(2t), ...)$ and $S_2 = \text{span}(\sin(t), \sin(2t), ...)$. Examine if $S_1 \oplus S_2$ is isomorphic to $S_1 + S_2$. (10 pts.)

(3) Consider the signal $s(t) = t^{\frac{1}{p}}$, $0 < p < 1$ and $t \in [-1, 1]$. Obtain a linear approximation to the signal $s(t)$ using the signals $f_1(t) = 1$ and $f_2(t) = t$ over the support $[-1, 1]$. Can you roughly trace the locus of $s(t)$ in the signal coordinate system as p is varied from 1 to ∞? (15 pts.)

PROBLEM 3: This problem has 2 parts.

(1) A random process $x[n]$ with zero mean and autocorrelation matrix $\mathbf{R}$ evaluated over lags from 0 to $m-1$ is applied to the input of a filter $H(z) = h_0 + h_1 z^{-1} + ... + h_{M-1} z^{-(M-1)}$. Let $\bar{h} = [h_0, h_1, ..., h_{M-1}]$ be the impulse response vector corresponding to $H(z)$. Determine the average power at the output in terms of $\bar{h}$ and $\mathbf{R}$. (10 pts.)

(2) Consider a first order auto-regressive process of the form $y[n] + ay[n-1] = w[n]$ with $0 < a < 1$ driven by a white noise $w[n]$ sequence with zero mean and variance σ_w^2. Obtain $\sigma_y^2 = E[y^2[n]]$ in terms of a and σ_w^2. (10 pts.)

PROBLEM 4: Consider a digital filter of the form $H(z) = \frac{1+az^{-1}}{(1-bz^{-1})(1-cz^{-1})^2}$, where $0 < a, b, c < 1$. Draw the signal flow graph with minimum delays and obtain the state variable representation. Show all your steps carefully. Indicate all the matrix elements clearly. (30 pts.)