

PROBLEM 1:

Consider the two channel filter bank.

- (a) Obtain the conditions on the synthesis filter banks to force the aliasing to zero. (1 point)
- (b) Let $H_0(z) = 1 + z^{-1}$ and $H_1(z) = 1 - z^{-1}$. Construct synthesis filter banks which ensure perfect reconstruction of the input signal. (1 point)

PROBLEM 2:

Obtain the Haar wavelet decomposition for the signal $f(t)$ using the Haar basis. Indicate the signal dimension at each subspace. Sketch the waveforms explicitly at each subspace. Obtain the reconstructed signal in functional form after nulling out any spike of $(1/8)$ th unit of time. Analyze using Fourier Transform. How much of energy is lost in the recovered signal?(8 points)

$$f(t) = \begin{cases} 2 & 0 \leq t < \frac{1}{4} \\ 1 & \frac{1}{4} \leq t < \frac{1}{2} \\ -1 & \frac{1}{2} \leq t < \frac{3}{4} \\ -2 & \frac{3}{4} \leq t < 1 \end{cases}$$

PROBLEM 3:

Consider the signal

$$x(t) = \begin{cases} 1 - |t| & \text{for } -1 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}.$$

Obtain the projection of $x(t)$ on V_0 and W_0 spaces of Haar multi resolution analysis. Is the projection shift invariant? (4 points)

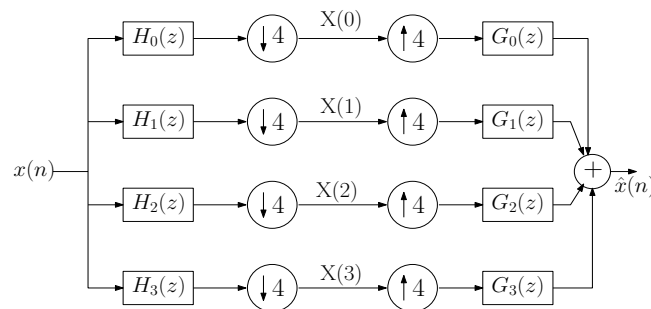
- (b) Compute $\sum_{n=-\infty}^{\infty} \phi(t - n)$. (1 point)

PROBLEM 4:

The normalized DFT of an N length sequence is defined as follows:

$$X(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi}{N} kn}$$

We wish to compute the normalized DFT $\{X(0), X(1), X(2), X(3)\}$ of a length 4 sequence using the 4 channel filter bank shown below:



- (a) Find the analysis filters $\{h_i(n)\}_{i=0}^3$ and synthesis filters $\{g_i(n)\}_{i=0}^3$ used to implement this filter bank. (8 points)
- (b) If the analysis filters are to be made causal, what is the delay introduced by the system? (2 points)

PROBLEM 5:

Problem 5.19 from the text P. P. Vaidyanathan (Multirate systems and filter banks). (4+3+3=10 points)

PROBLEM 6:

Problem 11.15 from the text P. P. Vaidyanathan (Multirate systems and filter banks). (5 points)

PROBLEM 7:

(a) Represent the Haar wavelet decomposition and reconstruction upto second scale as non uniform filter bank (i.e., decimation and upsampling rates are non uniform across different channels). What are the analysis filters $H_i(z)$ and synthesis filters $F_i(z)$ for this filter bank. (4 points)

(b) Using the multirate theory in the frequency domain, show that this filter bank achieves perfect reconstruction. (3 points)

(c) Test which of the special properties given below are satisfied by the filter bank. (2 points)

1. Strictly complementary
2. Power complementary
3. All pass complementary
4. Doubly complementary

(d) Are they Nyquist-m? (1 point)