

# Indian Institute of Science

E9-207: Basics of Signal Processing

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Mid Term Exam #1, Spring 2018

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**Name and S.R. No.:**

**Instructions:**

- Two sheets of paper are allowed.
- The time duration is 3 hrs.
- There are five main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and effort.
- Do not panic, do not cheat.
- Good luck!

Question No.	Points scored
1	
2	
3	
4	
5	
Total points	

PROBLEM 1: Examine if the following statements are true or false with correct reasoning. Random guessing or incorrect reasoning fetches zero credit. A statement is true if it is generic for all cases. A counter example is enough to make it false. All sub-parts of this problem carry equal credit.

1. Since downsampling and upsampling operations are not time invariant, all multi-rate systems that use downsamplers and upsamplers are non-LTI.
2. Let  $\underline{v}_i = (a_{i1}, a_{i2}, \dots, a_{iN})$  be a set of vectors for  $i = 1, 2, \dots, N$ . Let  $\underline{u}_i = (a_{1i}, a_{2i}, \dots, a_{Ni})$  be another set of vector  $i = 1, 2, \dots, N$ . We define two spaces  $\mathcal{V}$  and  $\mathcal{U}$  as  $\mathcal{V} = \text{Span}(\{\underline{v}_1, \underline{v}_2, \dots, \underline{v}_N\})$  and  $\mathcal{U} = \text{Span}(\{\underline{u}_1, \underline{u}_2, \dots, \underline{u}_N\})$ . We know that  $\dim(\mathcal{V}) < N$ . Then  $\dim(\mathcal{U}) > \dim(\mathcal{V})$ .
3. The inverse of a stable filter is also stable.
4. Let a signal  $s(t)$  be passed through a BIBO stable LTI system with impulse response  $h(t)$  to get the output  $y(t)$ . Let  $E_s$ ,  $E_h$  and  $E_y$  be the energies in  $s(t)$ ,  $h(t)$  and  $y(t)$  respectively. Then  $E_y \leq E_s E_h$ .
5. Let  $X(t)$  and  $Y(t)$  be two independent W.S.S processes. Their linear combination  $Z(t) = aX(t) + bY(t)$ ,  $a, b \in \mathbb{R}$  is also a W.S.S. process.

(25 pts.)

PROBLEM 2: This problem has two parts:

1. A discrete-time system with forcing function  $f[n]$  and output  $y[n]$  is represented using state variables  $u[n]$  and  $w[n]$  as

$$\begin{aligned}w[n+1] &= 2u[n] + 3f[n], \\u[n+1] &= w[n] + 2f[n], \\y[n] &= u[n] + 3w[n] + f[n].\end{aligned}$$

What are the modes of the system? What is the the transfer function of the system? What are the state space parameters ( $\mathbf{A}$ ,  $\underline{b}$ ,  $\underline{c}$ ,  $d$ ) of the system? (10 pts.)

2. Consider a cascade of two LTI systems  $A$  and  $B$  with impulse responses  $H_A(z) = \frac{1-z^{-1}}{(2+z^{-1})(1-3z^{-1})}$  and  $H_B(z) = \frac{1-3z^{-1}}{1-z^{-1}}$  respectively. Write down the time difference equations representing the systems  $A$  and  $B$ . Combine the two difference equations to obtain a time difference equation for the overall cascaded system. Compare the obtained equation with the overall impulse response of the cascaded system. (10 pts.)

PROBLEM 3: This problem has two parts:

1. Let  $\mathcal{S}$  be a finite dimensional vector space and  $\mathcal{V}_1 \subset \mathcal{S}$  and  $\mathcal{V}_2 \subset \mathcal{S}$  be two sub-spaces in  $\mathcal{S}$ . Show that  $\dim(\mathcal{V}_1 + \mathcal{V}_2) = \dim(\mathcal{V}_1) + \dim(\mathcal{V}_2) - \dim(\mathcal{V}_1 \cap \mathcal{V}_2)$ . (13 pts.)
2. Consider the following three signals given by

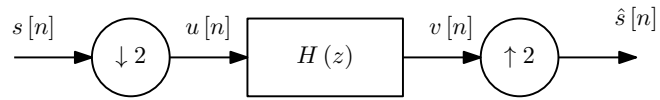
$$s_1(t) = \begin{cases} 1, & 0 \leq t \leq 0.25, \\ -1, & 0.25 < t \leq 0.75, \\ 1, & 0.75 < t \leq 1, \\ 0, & \text{otherwise,} \end{cases} \quad s_2(t) = \begin{cases} 1, & 0 \leq t \leq 0.5, \\ -1, & 0.5 < t \leq 1, \\ 0 & \text{otherwise,} \end{cases} \quad s_3(t) = \begin{cases} t - 0.5, & 0 \leq t \leq 1, \\ 0 & \text{otherwise,} \end{cases}.$$

Find an appropriate orthonormal basis for the signal space spanned by the three signals and represent the three signals as points in the signal space. What is the least squares approximation of the signal

$$s(t) = \begin{cases} \sin(2\pi t) & 0 \leq t \leq 1, \\ 0, & \text{otherwise,} \end{cases}$$

in the signal space. Plot the approximated signal as a function of time. (12 pts.)

PROBLEM 4: A signal  $s(t)$  with 60 Hz bandwidth and sampled at 600 Hz to obtain the samples  $s[n]$ . The signal  $s[n]$  is passed through following operations where  $H(z)$  is a low-pass filter. How do you choose the passband and stop band frequencies of  $H(z)$  such that the filter order is minimized and  $\hat{s}[n] = s[n]$ ? Sketch the frequency responses of the signal at various stages in the system.



(15 pts.)

PROBLEM 5: Consider the following fractional sampling rate converter that increases the sampling rate by a factor 1.5. Efficient architectures of the same circuit can be obtained in two ways:

1. Architecture-1: Represent  $H(z)$  using type-2 polyphase components  $\{E_0(z^3), E_1(z^3), E_2(z^3)\}$  to obtain an efficient architecture for the interpolation stage. Next, we represent each  $E_i(z)$  using type-1 polyphase components  $\{E_{i0}(z^2), E_{i1}(z^2)\}$  to obtain an efficient architecture for the decimation filters.
2. Architecture-2: Represent  $H(z)$  using type-1 polyphase components  $\{R_0(z^2), R_1(z^2)\}$  to obtain an efficient architecture for the decimation stage. Next, we represent each  $R_i(z)$  using polyphase components  $\{R_{i0}(z^3), R_{i1}(z^3), R_{i2}(z^3)\}$  to obtain an efficient architecture for the interpolation filters.

Express  $H(z)$  using the filters  $E_{ij}(z)$  from Architecture-1. Similarly, express  $H(z)$  using the filters  $R_{ij}(z)$  from Architecture-2. How are the filters  $E_{ij}(z)$  and  $R_{ij}(z)$  from the two architectures related? (15 pts.)