Indian Institute of Science

Linear and non-linear programming-1

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Home Work #2, Spring 2018

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, d = # days late

Assigned date: Feb. 20$^{\text{th}}$ 2018 **Due date:** Mar. 6$^{\text{th}}$ 2018 in class

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NOTATION

- $\mathbf{A}, \mathbf{D}, \mathbf{E} \in \Re^{m \times n}$.
- $\overline{x}, \overline{d}, \overline{x}_0, \overline{C}, \overline{C}^{\star} \in \Re^n$ and $\overline{b} \in \Re^m$.
- $\overline{x} \geq 0 \implies x_i \geq 0$ for all $i = 1, \ldots, n$.

PROBLEM 1: Prove that if the optimal value of a linear programming problem (LPP) occurs at more than one vertex of P_F (set of all feasible solutions), then it occurs at convex linear combination (clc) of these vertices. (10 pts.)

PROBLEM 2: Let $\overline{x}_0$ be an optimal solution of the LPP

$$\begin{aligned} \text{minimize} \quad & \overline{C}^{\mathrm{T}} \overline{x} \\ \text{subject to} \quad & \mathbf{A}\overline{x} = \overline{b} \\ & \overline{x} \geq 0 \end{aligned}$$

and let $\overline{x}^{\star}$ be any optimal solution when $\overline{C}$ is replaced by $\overline{C}^{\star}$. Then prove that

$$\left(\overline{C}^{\star} - \overline{C}\right)^{\mathrm{T}} \left(\overline{x}^{\star} - \overline{x}_0\right) \geq 0.$$

(10 pts.)

PROBLEM 3: Let $\overline{x} \in P = \{\overline{x} | \mathbf{A}\overline{x} = \overline{b}, \mathbf{D}\overline{x} \leq f, \mathbf{E}\overline{x} \leq g\}$ such that $\mathbf{D}\overline{x} = f$ and $\mathbf{E}\overline{x} = g$. Show that $\overline{d}$ is a feasible direction at $\overline{x}$ if and only if $\mathbf{A}\overline{d} = 0$ and $\mathbf{D}\overline{d} \leq 0$. (10 pts.)

PROBLEM 4: Consider the standard form polyhedron $\{\overline{x} | \mathbf{A}\overline{x} = \overline{b}, \overline{x} \geq 0\}$ and assume that the rows of matrix $\mathbf{A}$ are linearly independent.

(a) Suppose that two different bases lead to same basic solution, show that the basic solution is degenerate.
(b) Consider a degenerate basic solution. Is it true that it corresponds to two or more distinct bases? Prove or give counterexample.
(c) Suppose that a basic solution is degenerate. Is it true there exists an adjacent basic solution which is degenerate? Prove or give counterexample.

(10 pts.)

PROBLEM 5: Using simplex method

$$\begin{aligned} \text{maximize} \quad & 3x_1 + x_2 + 3x_3 \\ \text{subject to} \quad & 2x_1 + x_2 + x_3 \leq 2 \\ & x_1 + 2x_2 + 3x_3 \leq 5 \\ & 2x_1 + 2x_2 + x_3 \leq 6 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

(10 pts.)