E9-251: Signal Processing for Data Recording Channels

Home Work #3 (Due 23rd October 2012 in class)
Late Submission Policy: Points scored = Correct Points scored * $e^{-\#\text{days late}}$

Note:

- None of these problems require intense calculations.
- If you are doing the long way, you are probably on the wrong way.
- The architectures in the problems are for illustration. Do not confuse yourself by over-analyzing any extraneous variables beyond those required for the problem sets.

Problem 1:

Automatic Gain Control:

Consider the following block diagram Figure 1(a) of a simple discrete storage channel taking inputs $\{a_k\} \in \{-1,1\}$ with known impulse response $g[n]$. Additive noise $w[n]$ is shown. To simplify the case, there is no jitter. Let λ be the unknown gain of the channel.

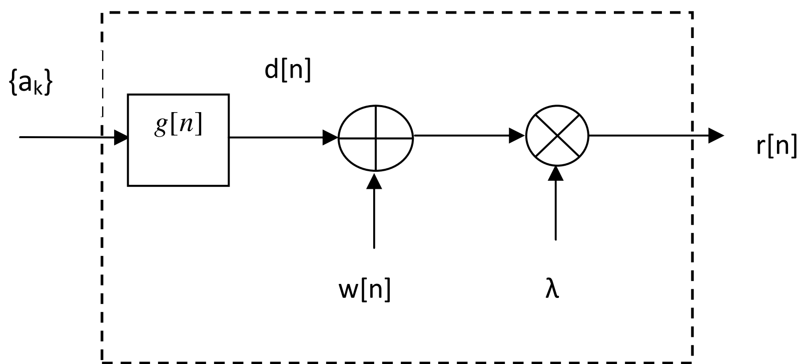

Fig 1(a): Channel model

We shall consider two architectures for gain control as follows.

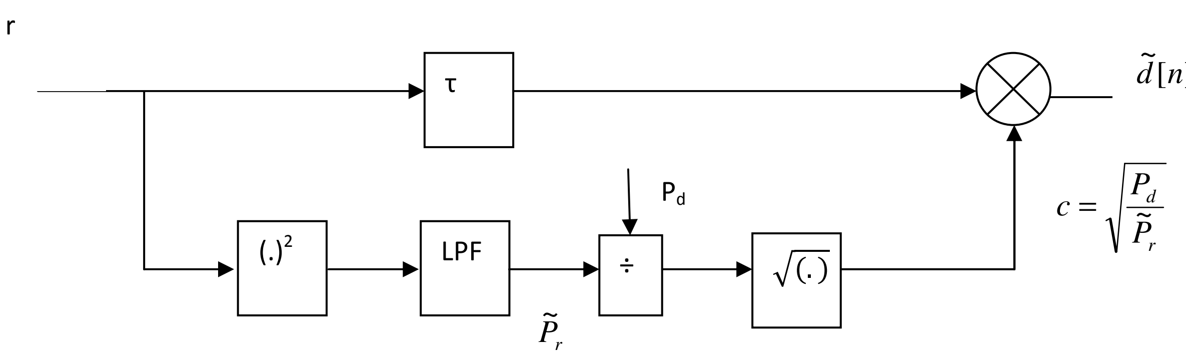

Fig 1(b): Non-data aided open loop architecture

The delay τ accounts for the effects of low pass filtering (LPF) used to remove any out of band noise. Let P_d denote the power of'd' and P_n denotes the noise power of 'w'. For convenience assume filtered data'd' is uncorrelated with 'w'.

1) Assuming that the estimate of r is accurate, i.e., $P_r = \tilde{P}_r$, show that steady state gain $\hat{c}$ for the open loop AGC in Fig 1(b) is given by $\hat{c} = \frac{1}{\lambda}\sqrt{\frac{P_d}{P_d + P_n}}$.
2) If there is no noise, what would you expect the steady state gain to be?
3) From (1), we see that the gain estimate has a suppression factor $\beta = \lambda\hat{c} = \sqrt{\frac{P_d}{P_d + P_n}}$ due to noise. If P_d/P_n = 20 dB, how large is the bias 1-β?

(10+5+5 pts)

Consider a second architecture that is data-aided using a closed loop feedback from detector

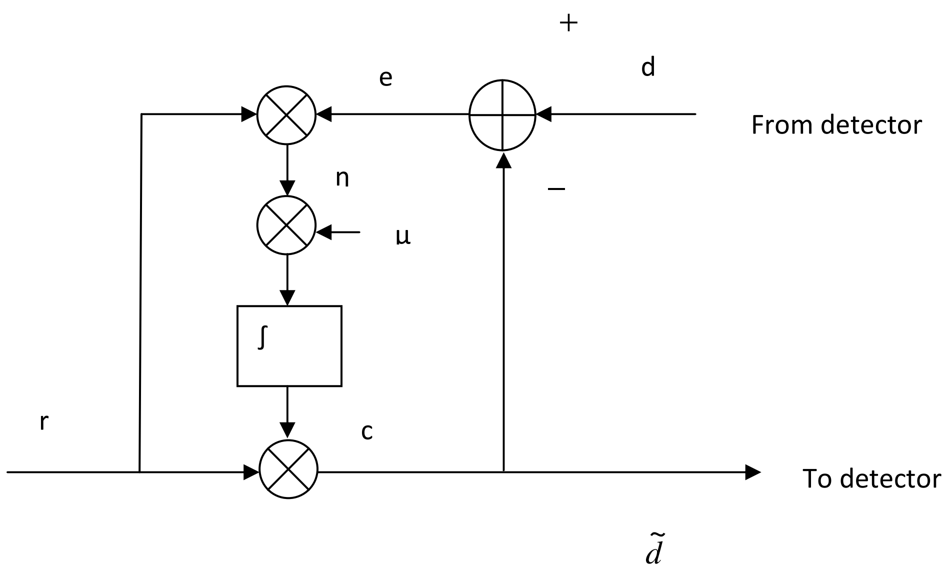

Fig 1(c): Closed loop data-aided AGC architecture

Denoting error e = $d - \tilde{d}$, we need to compute an estimate of the gain c that minimizes the mean square error (MSE) $E(e^2)$. The loop is driven so that η on an average heads to zero and there is an additional factor μ to control the loop stability/properties.

4) Show that the gain estimate as per MSE criterion is given by $\hat{c} = \frac{E(dr)}{E(r^2)}$.

(Hint: Carefully minimize the mean squared error.)

5) Using the channel model in Fig 1(a), prove that $E(dr) = \lambda P_d$ and $E(r^2) = \lambda^2 (P_d + P_n)$.
6) Denoting the gain suppression factor $\beta = \lambda \hat{c}$, what should be the ratio P_d/P_n so that β is less than 1 dB?

7) List 2 major merits/de-merits of the two architectures in Fig 1(b) and Fig 1(c).

(15+15+5+5 pts)

Problem 2:

Suppose we are filtering a discrete sequence x[n] through an FIR filter $H(z) = \sum_{i=0}^{N-1} h_i z^{-i}$ to produce a sequence y[n]. Assume that the bit budget is B bits for each of the filter coefficients, and they are represented using a sign bit, no more than 2 integer bits and B-3 fractional bits

1) If uniform quantization is done via rounding for each h_i, what is the overall quantization noise as a function of N and B?
2) Obtain an expression for signal-to-quantization noise in this case.
3) Is the noise at the output colored? Justify.

(10+15+5 pts)