

Indian Institute of Science

E9–252: Mathematical Methods and Techniques in Signal Processing

Instructor: Shayan G. Srinivasa

Homework #1, Fall 2017

Late submission policy: Points scored = Correct points scored  $\times e^{-d}$ ,  $d = \#$  days late

**Assigned date:** Aug. 28<sup>th</sup> 2017

**Due date:** Sept. 4<sup>th</sup> 2017 by end of the day

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**PROBLEM 1:**

Can convolution operator be expressed as an inner product? Justify.

**PROBLEM 2:**

Define inner products of vectors defined over a complex field  $\mathbb{C}$ . For complex vectors  $x$  and  $y$ , compute  $\langle x - y, x - y \rangle$  using the inner product defined. Derive the Cauchy-Schwarz inequality for complex vectors.

**PROBLEM 3:**

a) Let  $S_p = \{x : \|x\|_p \leq 1\}$ . Prove that  $S_p \subset S_{p+1}$ .

b) Prove that  $\lim_{p \rightarrow \infty} \mathcal{L}_p = \mathcal{L}_\infty$ .

**PROBLEM 4:** A function  $f : X \rightarrow \mathbb{R}$  is called convex if

$$f(\alpha x_1 + (1 - \alpha)x_2) \leq \alpha f(x_1) + (1 - \alpha)f(x_2) \quad \forall x_1, x_2 \in X \text{ and } \alpha \in [0, 1].$$

Examine if  $\text{norm}(\cdot)$  is a convex function.