Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

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Mid Term Exam#2, Fall 2016

Name and SR.No:

Instructions:

- You are allowed only 4 pages of written notes and a calculator for this exam. No wireless allowed.
- The time duration is 3 hrs.
- There are four main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and justification for partial credit.
- Do not panic, do not cheat.
- Good luck!

Question No.	Points scored
1	
2	
3	
4	
Total points	

PROBLEM 1: This problem has 3 parts.

Let $\mathcal{V}_n$ be the spaces generated from the Haar scaling function $\phi(2^n t - k)$, where k is any integer.

(1) What are the dimensions of $\mathcal{V}_n$ and $\mathcal{W}_n$ over the interval $t \in [0, 1)$? (5 pts.)

(2) With the usual notations as followed in the class, we have $\mathcal{V}_n = \mathcal{V}_0 \bigoplus_{k=0}^{n-1} \mathcal{W}_k$. Obtain the dimensions in $\mathcal{V}_0$ and $\{\mathcal{W}_k\}_{k=0}^{n-1}$ and use this to evaluate your answer for the dimension of $\mathcal{V}_n$. How does it compare to your result in sub part 1 of Problem 1? (10 pts.)

(3) Expand the signal $s(t) = 1 - t^2$ over the interval $t \in [-1, 1]$ using the Haar wavelet. (10 pts.)

PROBLEM 2: The following points $(2,0)^T$, $(3,-1)^T$, $(2,-2)^T$ and $(1,-1)^T$ occur with probabilities 1/8, 1/8, 3/8 and 3/8 respectively.

(1) Obtain the KL representation of the points. (8 pts.)

(2) If you were to reduce the points to 1D, how would you optimally represent them? Sketch the new 1D points carefully. (6 pts.)

(3) What fraction of the energy is lost by doing a dimensionality reduction in the previous step? (5 pts.)

(4) Suppose these four points correspond to four different classes, sketch the linear decision boundaries to separate the original set of points in 2D as well as in 1D after dimensionality reduction. Write down the equations of the boundaries explicitly. Are the linear decision boundaries unique? (6 pts.)

PROBLEM 3: Two students claimed to design a 3-channel perfect reconstruction filterbank. In one case, the student had a bank of analysis filters followed by down sampling rates of 3 in each branch. In another case, the down sampling rates were chosen to be 2, 3 and 6 in each of the three branches following the analysis filters. Justify if their claims are correct. Suppose a third student decided to go with a wavelet 3-channel filter bank using the Haar basis towards perfect reconstruction, what would you expect the down sampling rates to be at the analysis stage? Justify. (25 pts.)

PROBLEM 4: Consider a rectangular pulse $p(t)$ of amplitude A starting at the origin having a duration of T seconds. Treating time and frequency as random variables, compute the mean in time μ_t and in frequency μ_ω using a measure of the induced norm of the signal as a distribution. Justify if $\mu_t \leq \mu_\omega$ is true or false. (25 pts.)