Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

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Mid Term Exam#1, Fall 2016

Name and SR.No:

Instructions:

- Only four A4 pages/sheets of paper with written notes are allowed.
- The time duration is 3 hrs.
- There are four main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and justification for partial credit.
- Do not panic, do not cheat.
- Good luck!

Question No.	Points scored
1	
2	
3	
4	
Total points	

PROBLEM 1: This problem has 3 parts.

(1) (a) Is the inverse of a causal LTI system causal? Justify. (b) Is a finite duration signal always stable? Justify (5) pts.

(2) Let $\mathcal{V}$ be a vector space. Suppose $\mathcal{W}_1$ and $\mathcal{W}_2$ are subspaces of $\mathcal{V}$. Show that $\mathcal{W}_1 + \mathcal{W}_2$ is a subspace of $\mathcal{V}$ that contains $\mathcal{W}_1$ and $\mathcal{W}_2$. (10 pts.)

(3) Consider the space $\mathcal{V}$ spanned by the vectors $\mathbf{v}_1 = (1\ 2\ 1)^{\mathrm{T}}$, $\mathbf{v}_2 = (1\ 0\ 1)^{\mathrm{T}}$ and $\mathbf{v}_3 = (0\ -2\ 0)^{\mathrm{T}}$. Obtain the basis and dimension of $\mathcal{V}$ and $\mathcal{V}^{\perp}$. (10 pts.)

PROBLEM 2: This problem has 2 parts.

(1) Suppose the joint probability mass function (pmf) P_{XY} is uniform over all the three corners of an equilateral triangle whose base has vertices at $(-a, 0)$ and $(a, 0)$. Obtain the marginal pmfs. Are the random variables (a) independent (b) correlated? (10 pts.)

(2) Consider the random process $S(t) = A\cos(\omega t) + B\sin(\omega t)$, where ω is a constant and A and B are random variables. (a) What is the necessary condition for this process to be stationary? (b) If A and B are uncorrelated with equal variance, then $S(t)$ is wide sense stationary. Justify if the statement is correct. (15 pts.)

PROBLEM 3: This problem has 2 parts.

(1) If the low pass filter in a QMF bank is linear phase, the overall transfer function between the reconstructed output and input is guaranteed to be linear phase. Examine if this statement is true/false. Justify. (10 pts.)

(2) Suppose the low pass filter in a two-channel QMF bank is given by $H_0(z) = 2+6z^{-1}+z^{-2}+5z^{-3}+z^{-5}$, obtain a set of stable synthesis filters for perfect recovery. Sketch the polyphase implementation schematic. (15 pts.)

PROBLEM 4: This problem has 2 parts.

(1) Suppose a discrete time signal $x[n]$ is first upsampled by 13 followed by downsampling and upsampling by 3 and downsampling by 13 in the process of sampling rate conversions without any filtering operations in-between. Obtain the frequency domain response at the output after all your simplifications. (5 pts.)

(2) We need an efficient sampling rate conversion from 32 Ksamples/s to 48 Ksamples/s. From first principles, derive a fully efficient multirate architecture with all associated filters. Sketch the schematic of your multirate system. (20 pts.)