## Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

Instructor: Shayan G. Srinivasa Home Work #1, Fall 2015

Late submission policy: Points scored = Correct points scored  $\times e^{-d}$ , d = # days late

**Assigned date:** Aug 30<sup>th</sup> 2015 **Due date:** Sep 9<sup>th</sup> 2015 in class

PROBLEM 1: Provide an example of a 2D FIR filter with the following impulse response properties (a) non-causal (b) causal but unstable. Repeat this for the 2D IIR case. Justify your steps. (5 pts.)

PROBLEM 2: Problem 1.4.19 from the text Moon and Stirling (5 pts.)

PROBLEM 3: Problem 1.4.30 from the text Moon and Stirling.

(5 pts.)

PROBLEM 4: Random variables X and Y are uniformly distributed in the interval [0,1]. Assuming that X and Y are statistically independent, find the probability density function and the probability distribution function of a random variable Z = |X - Y|. (5 pts.)

PROBLEM 5: The power spectral density of a certain sequence x[n] is  $\frac{1}{a+b\cos(\omega)}$  for some non zero real constants a and b. Find the autocorrelation function. Suppose the autocorrelation function of a sequence x[n] behaves as  $r_{xx}(k) = \frac{1}{k}$  for time lags k > 1. What can you say on the power spectral density? (6 pts.)

PROBLEM 6: Suppose we are filtering a random sequence x[n] through a FIR filter  $1-az^{-1}$ , |a|<1. Let x[n] be a Bernoulli process such that P(x[n]=1)=p and P(x[n]=0)=1-p. Examine if this is a wide sense stationary process and ergodic in the mean. (7 pts.)

PROBLEM 7: Consider a collection of all  $n \times n$  matrices with real entries i.e.,  $M_n(R)$ . Is this a vector space? Justify.

- (1) Suppose we consider  $S := \{ \mathbf{X} \in M_n(R) : \det(\mathbf{X}) = 0 \}$ , examine if S is a subspace.
- (2) Suppose  $P, X \in M_n(R)$ . Let T be an operator such that  $T(X) = P^T X P$  for a fixed matrix P. Examine if T is linear.

(7 pts.)

PROBLEM 8: Consider the continuous time signal  $f(t) = \sum_{i=0}^{N-1} a_i u(t-i\frac{T}{8})$  where  $a_i$  is any real number, T is a time unit and N is a positive integer. Let  $\phi_1(t) = u(t) - u(t-\frac{T}{4})$  and  $\phi_2(t) = u(t) - 2u(t-\frac{T}{8}) + u(t-\frac{T}{4})$ .

- (1) Are  $\phi_1(t)$  and  $\phi_2(t)$  linearly independent?
- (2) Expand the signal f(t) in the  $\phi_1(t)$ - $\phi_2(t)$  plane after normalizing  $\phi_1(t)$  and  $\phi_2(t)$ . Interpret your results graphically.
- (3) Suppose a source emits  $\phi_1(t)$  and  $\phi_2(t)$  randomly with source probabilities p and 1-p respectively. Imagine a 2D cloud of uncorrelated Gaussian noise  $\mathcal{N}(\mathbf{0}, \sigma^2)$  acting in the  $\phi_1(t)$ - $\phi_2(t)$  plane. Determine the optimal linear decision boundaries to minimize the probability of misclassifying the signals  $\phi_1(t)$  and  $\phi_2(t)$ . Explicitly evaluate the probability of misclassification.

(10 pts.)