Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

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Mid Term Exam#2, Fall 2015

Name and SR.No:

Instructions:

- This is an open book, open notes exam. No wireless allowed.
- The time duration is 3 hrs.
- There are four main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and justification for partial credit.
- Do not panic, do not cheat.
- Good luck!

Question No.	Points scored
1	
2	
3	
4	
Total points	

PROBLEM 1: This problem has two parts.

(1) K sensors are placed in a field to take measurements from a source. Each sensor i is sensitive to a frequency band B_i, $1 \leq i \leq K$. The output of each sensor are samples $\{x_i[n]\}_{n=0}^{N-1}$ as shown in Figure 1. Assume that there is correlation between measurements across different sensors due to imperfections even though the frequency bands themselves are non overlapping.
Devise a technique to find the *dominant* frequency band based on the sensor data. Include all the inputs, intermediate variables and outputs in your procedure clearly. Prove the optimality of your solution from first principles. You can make any reasonable assumptions towards a solution on this problem by explicitly stating them. (20 pts.)

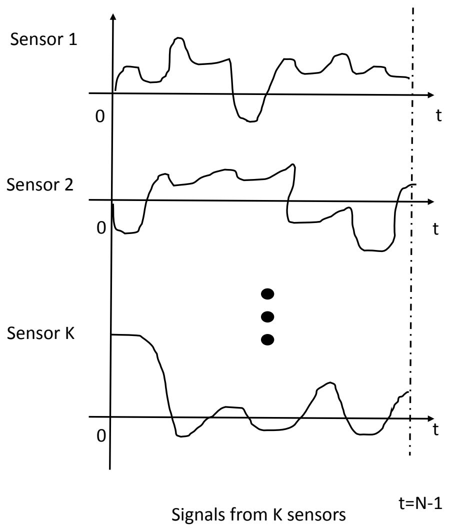

FIGURE 1. Samples from a multi-sensor array.

(2) Consider a 2π periodic piecewise continuous signal $f(t)$ with the existence of k derivatives. Let a_n and b_n denote the Fourier coefficients. Is $|a_n|, |b_n| \leq \frac{1}{\pi n^k} \int_{-\pi}^{\pi} |f^{(k)}(t)|dt$? (5 pts.)

PROBLEM 2: This problem has two parts.

(1) Suppose data is uniformly distributed inside a circle of radius a centered at the origin. What can you comment on the set of eigen vectors? Are they unique? (5 pts.)

(2) Consider a sequence of functions $f_n(t) = \frac{t^2+nt}{n}$ $\forall t \in \mathbb{R}$. (a) Examine the pointwise convergence and uniform convergence of $f_n(t)$ on $\mathbb{R}$. (b) If $f_n(t)$ is confined to an interval $[-a, a]$, what can you comment about its uniform convergence and L^2 convergence on the interval $[-a, a]$? Interpret your results graphically. (20 pts.)

PROBLEM 3: This problem has two parts

(1) Decompose the signal $f(t)$ using the Haar basis. Indicate the signal dimension at each subspace. Sketch the waveforms explicitly at each subspace. If you null out the subspace corresponding to the details at the highest resolution, what is your reconstructed signal in functional form? How much of energy is lost in the recovered signal?

$$f(t) = \begin{cases} 2 & 0 \le t < 0.25 \\ -4 & 0.25 \le t < 0.5 \\ 0 & 0.5 \le t < 0.75 \\ 1 & 0.75 \le t < 1 \end{cases}$$

(20 pts.)

(2) It is observed that a certain signal has a minimum resolution of $\frac{1}{5}$ time units. Suppose we are interested in a wavelet decomposition of the signal, what would be your choice of the scaling function and wavelet using Haar basis? What is the signal dimension in subspace $\mathcal{V}_n$ in this case? (5 pts.)

PROBLEM 4: Consider a J stage dyadic decomposition as shown in Figure 2(A). Let the low pass filter $H_0(z)$ and high pass filter $H_1(z)$ be first order FIR filters derived from the Haar basis. The filters are normalized to unit energy. This forms the analysis stage. We would like to have an equivalent representation as in Figure 2(B) with increasing decimation rates i.e., $D_0 \leq D_1 \leq ... \leq D_n$ as we progress from the top branch to the bottom branch in Figure 2(B) with one-to-one correspondence to branches in Figure 2(A).

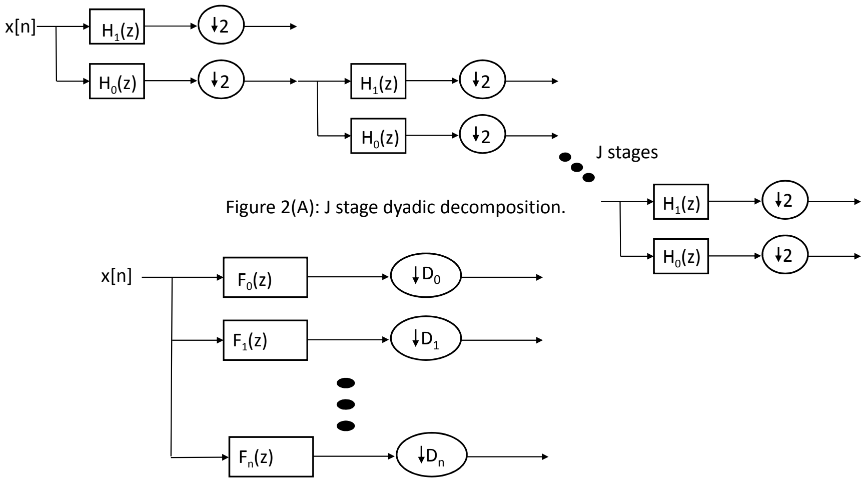

Figure 2(A): J stage dyadic decomposition.

Figure 2(B): Equivalent representation of Figure 2(A).

(1) What is n in Figure 2(B) in terms of J? Determine all the filters $F_i(z)$, $0 \leq i \leq n$ in terms $H_0(z)$ and $H_1(z)$. What are the values of the decimation rates D_i in Figure 2(B)? What frequency band does each branch correspond to in terms of normalized angular frequencies? (10 pts.)

(2) Obtain the architecture for the synthesis stage mirroring to the form in Figure 2(B) using upsamplers and synthesis filters. Explicitly compute the transfer functions of the corresponding synthesis filters indicating the filter orders. (10 pts.)

(3) What can you say about $\sum_{i=0}^{n} \frac{1}{D_i}$? (5 pts.)