Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

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Mid Term Exam#1, Fall 2015

Name and SR.No:

Instructions:

- Only four A4 sheets of paper with written notes on both sides are allowed .
- The time duration is 3 hrs.
- There are five main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and justification for partial credit.
- Do not panic, do not cheat.
- Good luck!

Question No.	Points scored
1	
2	
3	
4	
5	
Total points	

PROBLEM 1: Examine if the following statements are true or false with correct reasoning. Random guessing or incorrect reasoning fetches zero credit. A statement is true if it is generic for all cases. A counter example is enough to make it false. All sub-parts of this problem carry equal credit.

(1) Let $\bar{v}$ be the eigenvector for a square matrix A with a corresponding eigenvalue zero. $\bar{v}$ is also an eigenvector for A^n for integers $n > 1$.
(2) Consider the random process $s(t) = A\sin(\omega t)$, where A is Gaussian distributed with mean zero and variance σ^2. The frequency ω is deterministic. $s(t)$ is a stationary process.
(3) Upsampling of a non-zero discrete time signal by a factor $L > 1$ always causes images within the base band.
(4) Consider a discrete time signal $x[n]$ at D samples/s coded at c bits/sample with energy predominantly in the low pass region. Suppose we pass this signal through a two channel QMF bank so that the output of the analysis bank is coded at a bits and b bits per sample in the low and high frequency subbands respectively during transmission. We can achieve a compression in the data rate if $2c > a + b$.
(5) If an LTI system is causal, it is always stable.

(20 pts.)

PROBLEM 2: This problem has 3 parts.

(1) Let $\{\bar{v}_i\}_{i=1}^n$ be a collection of orthonormal vectors and $\{a_i\}_{i=1}^n$ be scalars.
Prove that $||\sum_{i=1}^{n} a_i\bar{v}_i||^2 = \sum_{i=1}^{n} |a_i|^2$. (8 pts.)

(2) Let the vector space $\mathcal{V}_1 \perp \mathcal{V}_2$.
(a) Prove that $\mathcal{V}_2$ is a subspace of $\mathcal{V}_1^{\perp}$. State the converse and verify if it is true.
(b) Prove that $\mathcal{V}_1 \cap \mathcal{V}_2 = \{\bar{0}\}$. (8 pts.)

(3) Consider the signals $f_1(t) = 1$ and $f_2(t) = t$ over $[-1, 1]$. Are they orthonormal? Obtain a closest linear signal approximation to $s(t) = t^2$ over $[-1, 1]$. Plot the approximate signal representation on a signal coordinate system. (9 pts.)

PROBLEM 3: Consider the structure shown in Figure 1, where $\mathbf{W}$ is the 3×3 DFT matrix. This is a three channel synthesis bank with three filters $F_0(z)$, $F_1(z)$ and $F_2(z)$. (For example, $F_0(z) = Y(z)/Y_0(z)$ with $y_1[n]$ and $y_2[n]$ set to zero.)

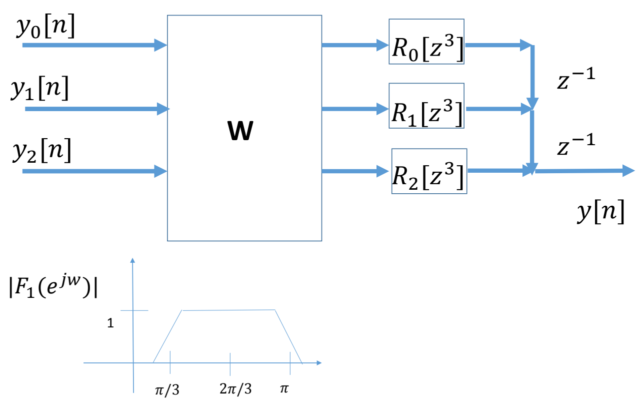

FIGURE 1. Three channel synthesis bank

(1) Assuming $R_0(z) = 1 + z^{-1}$, $R_1(z) = 1 - z^{-2}$, $R_2(z) = 2 + 3z^{-1}$, find an expression for the three synthesis filters $F_0(z)$, $F_1(z)$ and $F_2(z)$. (15 pts.)

(2) Let the magnitude of $F_1(z)$ be as shown above. Plot the frequency response of $|F_0(e^{j\omega}|$ and $|F_2(e^{j\omega}|$. (5 pts.)

PROBLEM 4: A certain sampling rate conversion system requires downsampling a signal at 100 Msamples/s to 40 Msamples/s. From first principles, derive a fully efficient architecture using downsamplers and expanders. Sketch the schematic of your multirate system. (25 pts.)

PROBLEM 5: The input to an LTI system is a WSS random process. Is the output WSS? Justify. You can assume that the LTI filter response is absolutely summable. (10 pts.)