

Indian Institute of Science

E9-252: Mathematical Methods and Techniques in Signal Processing

Instructor: Shayan G. Srinivasa

Home Work #4, Fall 2013

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, $d = \#$ days late

Assigned date: Oct 8th 2013

Due date: Oct 18th 2013 in class

PROBLEM 1: Consider the analysis/synthesis filter bank shown in Figure 1.

- (1) Let the analysis filters be $H_0(z) = 1 + 3z^{-1} + 0.5z^{-2} + z^{-3}$ and $H_1(z) = H_0(-z)$. Find causal stable IIR synthesis filters such that $\hat{x}[n]$ agrees with $x[n]$ with a possible delay and scale factor.
- (2) Let $H_0(z) = 1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}$ and $H_1(z) = H_0(-z)$. Find causal FIR synthesis filters such that $\hat{x}[n]$ agrees with $x[n]$ with a possible delay and scale factor.

(10 pts.)

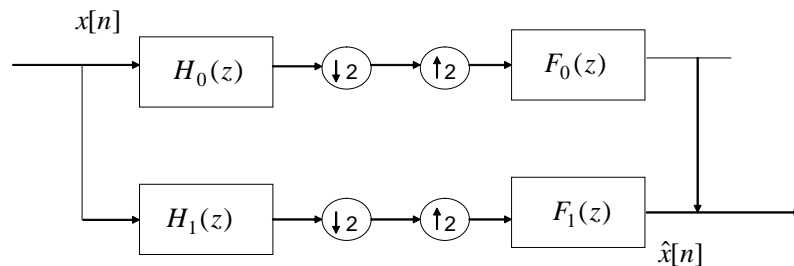


FIGURE 1. Two channel filter bank.

This problem is from the text by P. P. Vaidyanathan problem 4.28.

PROBLEM 2: Consider the following M channel multirate system as shown in Figure 2, which is essentially a QMF bank with additional transfer functions $C_k(z)$ introduced. We can imagine that $C_k(z)$ represents the amplitude and phase distortions introduced by the k^{th} channel.

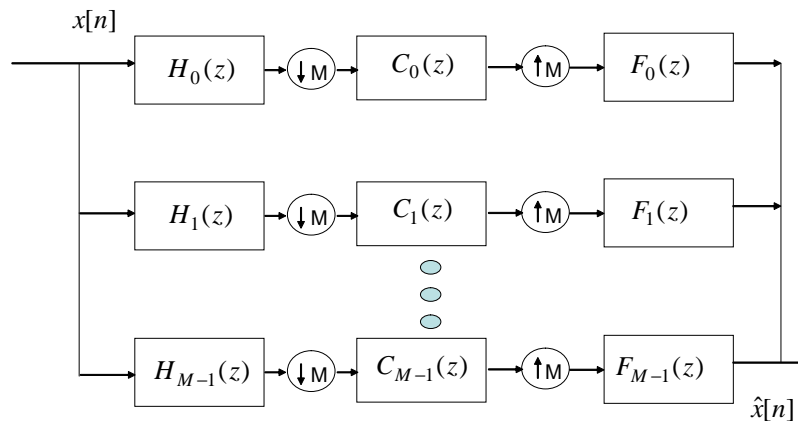


FIGURE 2. M-channel multirate system.

Assume throughout that the functions $H_k(z)$, $F_k(z)$ and $C_k(z)$ are rational and stable unless stated otherwise. Do not make any assumptions about the zeros of these transfer functions.

- (1) Let $H_k(z)$ and $F_k(z)$ be such that the system is ‘alias free’ in the absence of channel distortion $C_k(z)$. Let $C_k(z) \neq 1$. Determine a set of modified stable synthesis filters to retain alias-free property.
- (2) Repeat part (a) with the additional constraint ‘free from amplitude distortion’. Assume that $C_k(z)$ has no zeros on the unit circle.
- (3) Repeat part (a) by replacing ‘alias free’ with ‘perfect reconstruction’ everywhere. Assume now that $C_k(z)$ has no zeros on or outside the unit circle.

(15 pts.)

This problem is from the text by P. P. Vaidyanathan problem 5.23.

PROBLEM 3: Suppose we wish to design a 25-fold low-pass linear-phase interpolator. Let the input signal $x[n]$ be bandlimited to $|\omega| < 0.8$. Suppose the ripple specifications for $H(z)$ are $\delta_1 = 0.01$ and $\delta_2 = 0.005$.

- (1) Find the cutoff frequencies ω_p and ω_s for $H(z)$.
- (2) What is the filter order if a direct design is used?
- (3) Suppose a two-stage implementation is used, what are the filter orders? Show all the details of the frequency responses.
- (4) Compare the computational efficiencies for the two-stage and direct design if the input signal is a speech signal bandlimited to $4KHz$ under Nyquist sampling rate at the input.

(15 pts.)

PROBLEM 4: Four 2-D data points are given. They are $(3, 1)^T$, $(-3, -1)^T$, $(0, 5)^T$ and $(-5, 1)^T$. The probabilities of these points are $\{0.2, 0.3, 0.3, 0.2\}$ respectively.

- (1) What is the KL representation of these points?
- (2) Suppose we intend to retain only the dominant eigen component, what is the new representation of these points? What fraction of the signal energy is retained after dimensionality reduction?
- (3) How many linear hyperplanes are needed to classify the 4 signal points in 2-D? What would this correspond after dimensionality reduction?
- (4) Obtain the probability of error for signal classification using an optimal single hyperplane after dimensionality reduction. Show all your steps and reasoning.

(10 pts.)