Impact of Energy Quantization on the Performance of Current-Biased SET Circuits

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***Abstract*—The current-biased single electron transistor (SET) (CBS) is an integral part of almost all hybrid CMOS SET circuits. In this paper, for the first time, the effects of energy quantization on the performance of CBS-based circuits are studied through analytical modeling and Monte Carlo simulations. It is demonstrated that energy quantization has no impact on the gain of the CBS characteristics, although it changes the output voltage levels and oscillation periodicity. The effects of energy quantization are further studied for two circuits: negative differential resistance (NDR) and neuron cell, which use the CBS. A new model for the conductance of NDR characteristics is also formulated that includes the energy quantization term.**

***Index Terms*—Coulomb blockade, energy quantization, Monte Carlo technique, negative differential resistance (NDR), orthodox theory, single electron transistor (SET).**

I. Introduction

THE HYBRIDIZATION of single electron transistor (SET) with CMOS technology has attracted much attention [1], [2] as it promises novel functionalities that are very difficult to achieve either by pure CMOS or by pure SET approaches. Consequently, silicon SETs are appearing to be more promising than metallic SETs for their possible integration with CMOS. SETs are normally studied on the basis of the classical orthodox theory [3], where quantization of energy states in the island is completely ignored. Although this assumption greatly simplifies the physics involved, it is valid only when the SET is made of metallic island. As one cannot neglect the energy quantization in a semiconductive island, it is extremely important to study the effects of energy quantization on silicon SET circuit performance.

A current-biased SET (CBS) is an integral part of almost all hybrid CMOS SET circuits [1], [2] as it provides a unique triangular periodic output with monotonous increase of its gate voltage. In this paper, for the first time, the effects of energy quantization on the behavior of CBS-based circuits are studied through analytical modeling and Monte Carlo simulations. It is found that energy quantization has no impact on the gain of the CBS characteristics, although it changes the output voltage levels. The effects of energy quantization are further studied for two CBS-based pure SET circuits: negative differential resistance (NDR) [4] and neuron cell [5].[1] The hysteresis loop circuit, which exploits the NDR property, is also studied in this context. It is shown that the NDR conductance and the area of hysteresis loop degrade with increasing energy quantization and a compact model for the NDR slope is also formulated that explains this effect. On the other hand, it is found that energy quantization has no effect on the slope of the activation function characteristics of neuron-cell circuit; however, it changes the input- and output-voltage levels.

The energy quantization effects start to appear in SET characteristics when the dimension of the SET island becomes comparable to the Fermi wavelength of the electrons, which is inversely proportional to the free-electron density. Therefore, the amount of energy quantization in the SET island depends on several properties of the islandlike size, shape, material, doping concentration, property of the tunnel barrier, etc. In this paper, we have not attempted to deal with the complex quantum physics involved in obtaining a definite value of energy gap between two successive energy levels (ΔE) for any particular device geometry. Here, ΔE is treated as an *electrical parameter* so that we can study the effects of energy quantization when it is *gradually* introduced into a metallic island without detailing how to obtain the exact value of ΔE for the structure. The problem of developing analytical expressions for the quantum physics involved in ΔE itself is a complicated work and *not* the objective of this paper. However, the range of values considered in all our simulations, as well as our development of analytical model, is quite practical, as they are in the range of quantized energy steps analogous to the square-potential-well problem (if we consider that energy quantization is solely due to the island geometry) which are often encountered in low-dimensional physics. It should be noted that, in this paper, we use parabolic potential well so that all ΔE are equal to avoid complication.

II. Results and Discussions

In this paper, the widely accepted single electron device simulator SIMON [6] is used to comprehend the effects of energy quantization. So far, SIMON has been used extensively

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[1] There might be many other CBS-based pure SET circuits; however, in this paper, we concentrate only on these two. The effects of energy quantization could be studied in the similar fashion for the other circuits.

to study SET circuits that obey the orthodox theory (metallic island). To our best knowledge, in this paper, SIMON simulator (version 2.0) has been used for the first time in order to analyze energy quantization effects on the performance of CBS circuits. SIMON simulates the discrete energy levels ΔE of the island as Lorentzian-shape functions, whose height H and width W parameters are related to the energy dependence of the transmission probability, which depends on the tunnel-barrier resistance [7]–[9] and need to be calibrated manually.

One might conceptualize a metallic SET to be equivalent to a nonmetallic one in which the energy states of the island extend from lower bound $E_{\min} \rightarrow -\infty$ to upper bound $E_{\max} \rightarrow +\infty$ with the energy gaps between successive energy states $\Delta E \rightarrow 0$. In order to study the energy quantization effects, we first simulate a SET with metallic (continuous energy spectrum) island for a particular set of device parameters (C_G, $C_T \sim aF$, and $R_T \sim M\Omega$, where C_G is the gate capacitance, C_T is the tunnel-junction capacitance, and R_T is the tunnel-junction resistance). Then, for the same set of device parameters, we simulate a nonmetallic SET with discrete states, where $E_{\min} = -1$ eV, $E_{\max} = 1$ eV, and $\Delta E = 0.01$ meV. As the magnitudes of $E_{\min}$ and $E_{\max}$ are much larger and ΔE is much smaller than the charging energy (~160 meV) of the SET, we can expect that such a device should behave as metallic SET if the W and H parameters are properly tuned. After exhaustive simulations, we have found that, for $H = 0.1$ and $W = 0.01$, the I–V characteristics of the nonmetallic SET with discrete energy states completely superimposes over the characteristics obtained from the metallic SET. It is further observed that these values of H and W are almost independent of device capacitances and resistances as long as their values lie in the range of aF and $M\Omega$, respectively. Using these calibrated magnitudes of H and W and keeping $E_{\max}$ and $E_{\min}$ constant, the value of ΔE has been gradually increased in order to simulate the effects of energy quantization on SET device and circuit performances.

When energy quantization is introduced, the net change in energy (ΔF) of the electrons during tunneling becomes the sum of electrostatic energy contributed by Gibb's free energy (addition energy) as well as the energy gaps between the quantized energy levels (excitation energy). Consequently, including the quantization term ΔE into the expression for the ΔF for $n \rightarrow n+1$ transition, we obtain [10], [11]

$$\begin{Bmatrix} \Delta F_{s,i} \\ -\Delta F_{i,s} \end{Bmatrix} = \frac{q}{C_\Sigma}\left[C_T V_{\mathrm{DS}} + C_G V_{\mathrm{GS}} - nq \mp \frac{q}{2}\right] - \begin{Bmatrix} (n+1) \\ n \end{Bmatrix} \Delta E \tag{1}$$

$$\begin{Bmatrix} \Delta F_{i,d} \\ -\Delta F_{d,i} \end{Bmatrix} = \frac{q}{C_\Sigma}\left[(C_G + C_T) V_{\mathrm{DS}} - C_G V_{\mathrm{GS}} + nq \mp \frac{q}{2}\right] - \begin{Bmatrix} n \\ (n+1) \end{Bmatrix} \Delta E. \tag{2}$$

Throughout this paper, in multiple equations like (1) and (2), the upper term in the left-hand side equates the upper

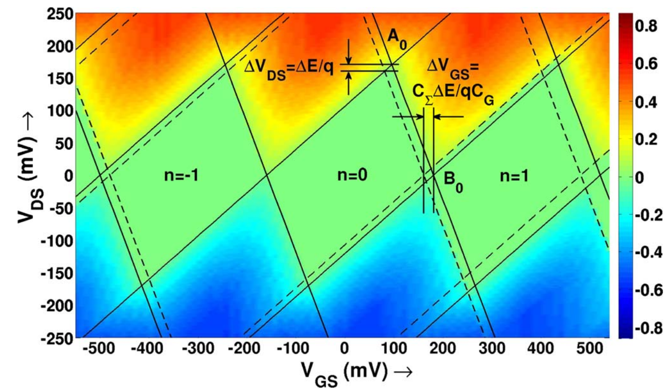

Fig. 1. Influence of energy quantization ΔE on $V_{\mathrm{DS}} - V_{\mathrm{GS}}$ surface characteristics (drain–current contour) of a SET. The bold and broken lines enclose the Coulomb blockade regions for the quantized SET (with discrete energy states) and the metallic SET (with continuous energy states). Simulated for $R_T = 1$ MΩ, $C_G = 0.5$ aF, $C_T = 0.25$ aF, and temperature $T = 1$ K.

symbol sequence on the right-hand side and vice-versa. Here, q is the elementary charge, C_Σ is the total island capacitance with respect to ground (equal to $C_G + 2C_T$), n denotes the number of discrete energy states in the island, and $\Delta F_{\mathrm{initial,final}}$ denotes the net free-energy change for the electron tunneling from "initial" to "final" which may be any of the source "s," island "i," or drain "d" regions.

Fig. 1 shows the influence of energy quantization on the $V_{\mathrm{DS}} - V_{\mathrm{GS}}$ surface characteristics (drain–current contour plot) as obtained from the SIMON simulation for $\Delta E = 10$ meV. In the same figure, the earlier four linear equations (1) and (2) are plotted with $\Delta F = 0$ for different n, and they encompass the four sides of the so-called "Coulomb blockade parallelogram." It is evident from Fig. 1 that increasing energy quantization increases the area of the Coulomb blockade region as indicated by the region enclosed by the bold lines being more than that by the broken lines. By solving those four linear equations, the coordinates of the intersection points A_n and B_n (Fig. 1) are found to be

$$A_n \equiv \left[\frac{nq}{C_G} + \frac{q}{2C_\Sigma} + \frac{\Delta E}{q}\left(n + \frac{C_G + C_T}{C_G}\right), \frac{q}{C_\Sigma} + \frac{\Delta E}{q}\right] \tag{3}$$

$$B_n \equiv \left[(n-1)\frac{q}{C_G} + n\frac{\Delta E}{q}, 0\right]. \tag{4}$$

Using (3) and (4), the increase of Coulomb blockade area due to energy quantization could be found as

$$\Delta\text{area} = \frac{\Delta E}{q}\left[q\left(\frac{1}{C_G} + \frac{1}{C_\Sigma}\right) + \frac{\Delta E}{q}\right]. \tag{5}$$

As the current contour plot (Fig. 1) can also be treated as input–output characteristics of a CBS, it can be said that energy quantization increases the voltage level by an amount $\Delta V_{\mathrm{DS}} = \Delta E/q$ and the periodicity by an amount $\Delta V_{\mathrm{GS}} = C_\Sigma \Delta E / q C_G$ of a CBS output. At the same time, as the slope

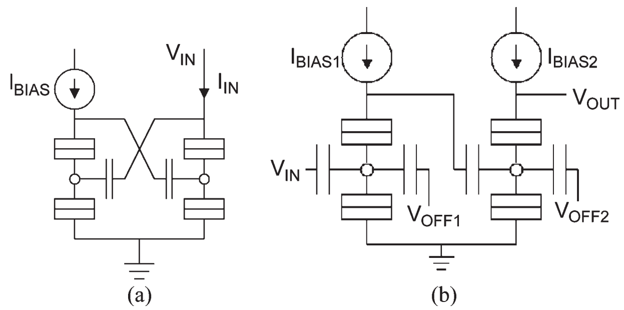

Fig. 2. Circuit schematics of (a) NDR [4] and (b) neuron cell [5].

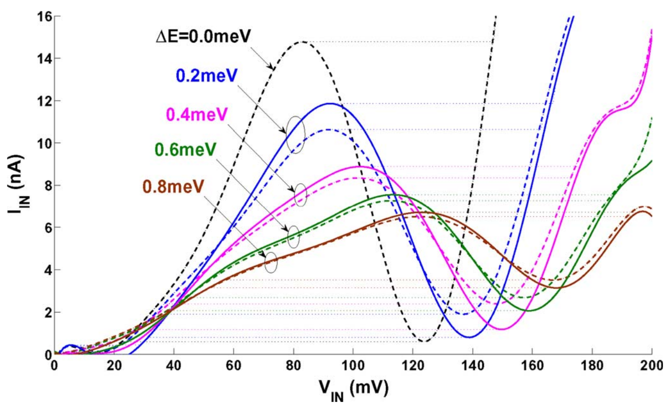

Fig. 3. Influence of energy quantization ΔE and temperature on $I_{\rm IN}-V_{\rm IN}$ characteristics of NDR circuit. Bold and the broken lines represent the plots at temperatures 80 K and 40 K, respectively. The minor dotted lines represent the path followed by the circuit during operation, giving rise to the hysteresis shown in Fig. 5. Simulated for $I_{\rm BIAS} = 5$ nA, $R_T = 1$ MΩ, $C_G = 0.5$ aF, $C_T = 0.25$ aF, and $C_L = 1$ fF.

$(dV_{\rm DS}/dV_{\rm GS})$ of the equations $\Delta F = 0$ does not depend on ΔE, it is inferred that the differential gain of CBS is independent of energy quantization. It is noteworthy that the increase of Coulomb blockade area due to energy quantization has been experimentally demonstrated by Saitoh and Hiramoto [12]. All these observations will now be used to analyze the impact of energy quantization on CBS-based circuits like NDR and neuron cell.

It is finally noteworthy that SIMON does not use (1) and (2) to simulate an island with discrete energy states. SIMON uses Lorentzian function to capture energy quantization effect and consider the addition energy and excitation energy as two distinct processes. Although, in this paper, we use (1) and (2) to explain quantization effects, which combines these two energies, the perfect superimposition of the bold and broken lines with the simulated Coulomb blockade region (the contour plot) in Fig. 1 validates our approach for studying the influence of energy quantization.

CBS is an integral part of almost all CMOS SET hybrid circuits. However, due to the lack of proper SET–MOSFET cosimulation framework, in this paper, we will limit our discussions to pure SET circuits that use CBS. First, we discuss the effect of energy quantization on NDR (and respective hysteresis circuit), which was proposed by Mahapatra *et al.* [4], [13].

Fig. 2(a) shows the schematic of the CBS-based NDR circuit, and Fig. 3 shows the influence of energy quantization on the $I_{\rm IN}-V_{\rm IN}$ characteristics of the circuit. It indicates that increasing energy quantization ΔE as well as increasing temperature decrease the dynamic range as well as the slope of the NDR region of the characteristics. In addition, the peak position of the NDR is shifting toward the higher $V_{\rm IN}$ values as the x-coordinate of A_n increases with energy quantization. From the current contour plot (Fig. 1), one can see that, for a constant $I_{\rm BIAS}$, the output voltage of a CBS increases with ΔE. At the same time, the drain current of a voltage-biased SET decreases due to energy quantization as it increases the effective resistance. Due to these two effects, the NDR region decreases with increasing energy quantization.

Now, considering only the most probable electron-tunneling events ($0 \leftrightarrow 1$ transitions) in the SETs, the net current may be written as

$$I_{\rm IN} = \frac{1}{R_T V_{\rm IN}}\left[\frac{C_G + C_T}{C_\Sigma}V_{\rm IN} - \frac{C_G}{C_\Sigma}V_{\rm OUT} + \alpha\right] \times \left[\frac{C_T}{C_\Sigma}V_{\rm IN} + \frac{C_G}{C_\Sigma}V_{\rm OUT} - \alpha\right] \quad (6)$$

where $\alpha = (q/2C_\Sigma + \Delta E/q)$. Equation (6) is differentiated with respect to $V_{\rm IN}$ to obtain

$$R_T C_\Sigma^2 \frac{\partial I_{\rm IN}}{\partial V_{\rm IN}} = \frac{\partial V_{\rm OUT}}{\partial V_{\rm IN}}\left[C_G^2 + \frac{2\alpha C_G C_\Sigma}{V_{\rm IN}} - \frac{2C_G^2 V_{\rm OUT}}{V_{\rm IN}}\right] + \frac{\alpha^2 C_\Sigma^2}{V_{\rm IN}^2} + \frac{C_G^2 V_{\rm OUT}^2}{V_{\rm IN}^2} - \frac{2\alpha C_G C_\Sigma V_{\rm OUT}}{V_{\rm IN}^2} + C_T(C_G + C_T). \quad (7)$$

Replacing $V_{\rm GS} = V_{\rm OUT}$ and $V_{\rm DS} = V_{\rm IN}$ in (2) and differentiating it, we get

$$\frac{\partial V_{\rm OUT}}{\partial V_{\rm IN}} = \frac{C_G + C_T}{C_G}. \quad (8)$$

Using (1), (2), and (8) in (7), we get the compact form

$$\frac{\partial I_{\rm IN}}{\partial V_{\rm IN}} = \frac{\alpha^2}{R_T V_{\rm IN}^2} = \frac{1}{R_T V_{\rm IN}^2}\left(\frac{q}{2C_\Sigma} + \frac{\Delta E}{q}\right)^2. \quad (9)$$

Putting $\Delta E = 0$, (9) reduces to

$$\frac{\partial I_{\rm IN}}{\partial V_{\rm IN}} = \frac{1}{R_T V_{\rm IN}^2}\left[\frac{q}{2C_\Sigma}\right]^2 \quad (10)$$

which is the form observed in metallic SETs as predicted by the orthodox theory.

Fig. 4 shows the validation of the analytical model (9). On close observation, it can be observed that the model starts to deviate from the simulated results when the current $I_{\rm IN}$ reaches the maximum. This is due to the fact that, in this peak region, the output of CBS becomes higher than q/C_Σ, where it is not enough just to consider "$0 \rightarrow 1$" transition. It should be noted that, in this paper, we obtain a good match using MIB model which is based on the orthodox theory. This has been possible

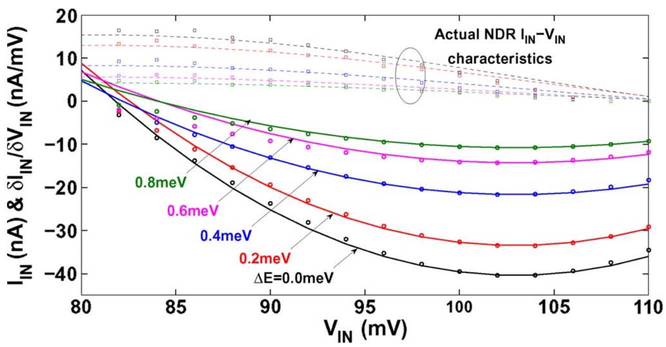

Fig. 4. Validation of the analytical model with the simulated results. The symbols denote the simulated results. The broken lines indicate the magnified NDR region of the smoothed simulated results of $I_{\rm IN}-V_{\rm IN}$ characteristics (Fig. 3) while the solid lines indicate the analytical model. Simulated for $I_{\rm BIAS} = 0.1$ nA, $C_G = 0.5$ aF, $C_T = 0.25$ aF, $R_T = 1$ MΩ, and $C_L = 1$ fF.

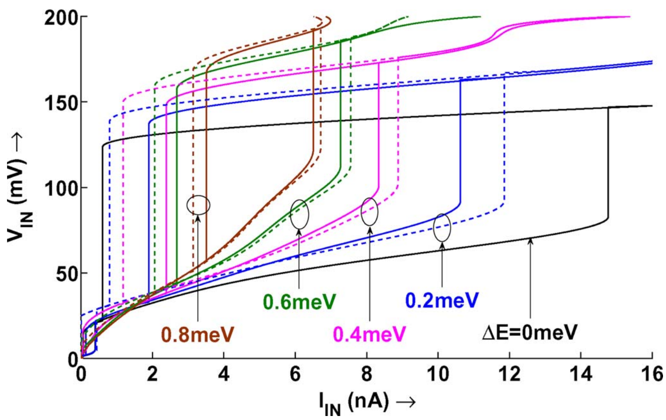

Fig. 5. Influence of energy quantization ΔE on the hysteresis of the NDR characteristics in Fig. 3. Simulated for $I_{\rm BIAS} = 5$ nA, $R_T = 1$ MΩ, $C_G = 0.5$ aF, $C_T = 0.25$ aF, and $C_L = 1$ fF.

as the ΔE is very small as compared to q^2/C_Σ and acting as small perturbation. In addition, these equations does not contain any temperature term, which means that they are valid for the temperature range $T < q^2/40k_BC_\Sigma$, where k_B is Boltzmann's constant.

The NDR circuit can also be used as hysteresis loop circuit [13], and the effect of energy quantization on the loop area is shown in Fig. 5. As the NDR region degrades with ΔE, the area of the hysteresis loop also reduces.

Connecting two CBS circuits in cascaded mode, one gets the single-input neuron cell [5], as shown in Fig. 2(b). The effects of energy quantization on the neuron cell are shown in Fig. 6. As the neuron cell uses two cascaded CBS and as the differential gain of CBS characteristics is independent of energy quantization, the rising slope of $V_{\rm IN}-V_{\rm OUT}$ characteristics are unaffected by ΔE. However, the voltage levels change quite significantly with energy quantization, since the output of CBS changes with energy quantization.

III. Conclusion

In this paper, using analytical models and Monte Carlo simulation, the effects of energy quantization on CBS circuits are studied. It is observed that energy quantization in the

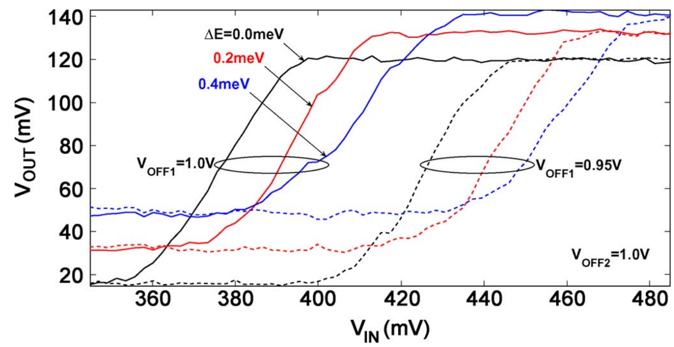

Fig. 6. Influence of energy quantization ΔE on $V_{\rm OUT}-V_{\rm IN}$ characteristics of the CBS-based single-input neuron-cell circuit. Simulated for $I_{\rm BIAS1} = I_{\rm BIAS2} = 5$ nA, $R_T = 1$ MΩ, $C_G = 0.5$ aF, $C_T = 0.25$ aF, and $C_L = 1$ fF.

SET island mainly increases the Coulomb blockade region and periodicity of the Coulomb blockade oscillation characteristics. The effects of energy quantization are further studied for two circuits: NDR and neuron cell, which use CBS. New analytical models are developed to explain the energy quantization effects on the performance of CBS circuits. The impact of this paper lies in the fact that the future of SETs completely depends on the successful integration of the CMOS and the SET technologies which is possible only with semiconductor SETs, where the quantization effects cannot be ignored at all. This paper reports the energy quantization effects on CBS circuits which are an integral part of CMOS SET hybrid circuits.

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