

Indian Institute of Science

Quantum Information Theory

Instructor: Shayan Srinivasa Garani

Home Work #1, Fall 2022

Late submission policy: Points scored = Correct points scored $\times e^{-d}$, $d = \#$ days late

Assigned date: Sep. 5th, 2022

Due date: Sep. 18th, 2022, 11:59 pm

PROBLEM 1: Consider a discrete memoryless binary *asymmetric* channel with the following conditional error probabilities i.e., $P(r = 1|s = 0) = p$ and $P(r = 0|s = 1) = q$. The random variables r and s stand for receiver and sender. (a) Obtain a closed form expression for the channel capacity. (b) Obtain the error probability as a function of the channel parameters using a three-bit majority voting rule under the mapping $0 \rightarrow 000, 1 \rightarrow 111$. How does the error exponent scale with serial concatenations of the code? (15 pts.)

PROBLEM 2: Solve the following exercise problems in Mark Wilde's book: 3.5.4, 3.5.5, 3.5.6, 3.5.13. (10 pts.)

PROBLEM 3: Consider the Klauder-Glauber-Sudarshan states also called *coherent* states, given by $|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$, $\alpha \in \mathbb{C}$. $|n\rangle$ represents the number counting states of the Hamiltonian $H = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2})$. Here, \hat{a}^\dagger and \hat{a} represent the photon creation and annihilation operators with the property that $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$.

- (1) Prove that the states form a complete basis i.e., $\sum_{n=0}^{\infty} |n\rangle\langle n| = \mathbb{I}$.
- (2) Are the states orthogonal? Justify.
- (3) Using the Dirac notation learnt in the class, evaluate the average and variance of the photon number in the coherent state.
- (4) Do these states satisfy the uncertainty principle? Justify with all the details from first principles. (25 pts.)

PROBLEM 4: Prove the following results:

- (1) Quantum no-deletion theorem. We discussed the theorem statement in the class.
- (2) Suppose parties A and B shared an ebit $|\Phi^+\rangle^{AB}$ with no communication between them. Show that if a universal quantum cloner existed, then A could signal to B at a speed greater than the speed of light by exploiting the ebit. (15 pts.)

PROBLEM 5: A source emits bits in such a way that there are no two consecutive ones occurring anywhere within the sequence. What can you comment on the size of the *typical set* as function of n realizations of such a random sequence? Empirically generate such random sequences from your source model. Plot the sample entropy as a function of n . What do you observe for large n ? (10 pts.)