Indian Institute of Science

E9: 252 Mathematical Methods and Techniques in Signal Processing

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Mid Term Exam, Spring 2022

Name and SR.No:

Instructions:

- This is an open book, open notes exam.
- There are four main questions. None of them have negative marking.
- Attempt all of them with careful reasoning and justification for partial credit.
- There is absolutely no collaboration with any human or a bot.
- This is a take home exam. Assigned on March 26th, 2022, 11:59 pm. Turn in by March 28th, 2022 11:59 pm.
- Do not panic, do not cheat, good luck!

Question No.	Points scored
1	
2	
3	
4	
Total points	

PROBLEM 1: A signal $x[n]$ with a sampling rate of 20 KHz is to be downsampled by a factor $M = 50$ to produce a 400 Hz signal. The passband of the signal is from 0-150 Hz. The band extending from 150-200 Hz is the transition band. The passband and stop band ripples of 1e-3 are desired. Design the best sampling rate conversion system for this specification. Your solution must be the *optimal* one in terms of computational complexity. Justify all the details of your work. (35 pts.)

PROBLEM 2: A signal sampled at 50 Mbps must be upsampled to 70 Mbps followed by filtering through a filter $H(z)$. How do you accomplish this through an efficient architecture? Suppose the input signal has a spectrum $X(e^{j\omega})$, what is the spectrum at the output of the convertor? Show all the details of your work carefully. (20 pts.)

PROBLEM 3: Consider a biased Bernoulli source emitting a sequence of binary digits ± 1. The probability of digit '1' is p. This discrete time sequence is filtered through (a) an IIR filter $H(z) = \frac{a}{1+bz^{-1}}$, and (b) an FIR filter $H(z) = a + bz^{-1}$. Compute the power spectral density at the output of the filters. Suppose a and b take values within $[-1, 1]$, comment on the nature of the power spectral density i.e., entirely low pass or high pass etc. (15 pts.)

PROBLEM 4: This problem has four parts:

(1) Gaussian random samples $\mathcal{N}(0, \sigma^2)$ are quantized through a ceil operator. Compute the sample mean post quantization. Verify your analysis via simulations. (5 pts.)

(2) Is it possible that the autocorrelation matrix of a certain random process is not symmetric? If so, justify the nature of the random process through an example. (5 pts.)

(3) Obtain a state space representation for $H(z) = \frac{1+z^{-1}}{1+0.5z^{-1}+2z^{-2}}$. Show all your steps analytically. (15 pts.)

(4) A student filtered a random discrete time sequence from a source through an unknown LTI filter and analyzed its spectrum. He always observed a dc-component at the output in several runs of his experiments. What inferences can be drawn on the spectra of the source and the spectrum of the filter? (5 pts.)